

General Dynamics.

Higher Mechanics. By Prof. H. Lamb.
Pp. x+272. (Cambridge: At the University Press, 1920.) Price 25s. net.

WE have here, as was to be expected from its author, an excellent statement and explanation of the principal theorems of what may properly be called higher dynamics. Prof. Lamb's title is "Higher Mechanics," which is in accordance with the usage which obtained before the appearance of Thomson and Tait's "Natural Philosophy." In his preface to the "Principia," Newton says: "Mechanicam vero duplicem veteres constituerunt, rationalem, quæ per demonstrationes accurate procedit, et practicam." Thus Newton uses the word "mechanica" qualified by the adjective "rationalis" in the sense of "scientia motuum qui ex viribus quibuscunque resultant," the science of the motions of bodies. The point is not one of great importance, but we prefer with Thomson and Tait the name "dynamics" for the whole science of rational mechanics.

The book begins with chapters on the kinematics of a rigid body, in which degrees of freedom, displacements of a rigid body, and theorems regarding these are very clearly explained and illustrated. Then follows a chapter on statics, in which the usual theorems are set forth, with a short but adequate account of the theory of screws. A treatment of moments of inertia is given which, perhaps, might with advantage have been fuller. Binet's theory of plane quadratic moments is included, while his theorem that the principal axes of inertia at any point P are the normals to the three quadrics which are confocal with the central ellipsoid of gyration, and which intersect at P, is not attributed to its author. The subject of plane quadratic moments does not seem of any but a purely mathematical interest, and one rather grudges the page devoted to it in this brief chapter.

After a chapter on instantaneous motions, we are led on to the equations of motion of a rigid body, with reference to a system of moving axes, which finally, when the body turns about a fixed point, is identified with the principal axes of moment of inertia passing through the fixed point. In connection with the equations of motion for this case—Euler's equations—which are of the form

$$A\dot{p} - (B - C)qr = L,$$

the usual statement is made that the equations may be written:

$$A\dot{p} = L + (B - C)qr,$$

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and that then the quantities typified by $(B - C)qr$ are referred to as the components of a "centrifugal couple." This couple is quite rightly designated "fictitious"; but, apart from the lack of reality which afflicts it, its introduction seems undesirable. To speak of these quantities as "centrifugal couples" seems a perfectly artificial, unphysical, and almost unintelligible mode of regarding the matter. Centrifugal couples are worse than centrifugal forces, those unrealities against which Tait used to fulminate vehemently.

What one wants to convince the student here is that $A\dot{p}$ is not the whole rate of growth of angular momentum about the fixed axis, with which the principal axis at the instant coincides, and must be supplemented by the rate of growth $-(B - C)qr$ which exists in consequence of the motion of the axes. The whole rate of growth is $A\dot{p} - (B - C)qr$, and there is no ray of light thrown on the subject by carrying the second part from its proper place to the other side of the equation and calling it a couple. The use of "centrifugal couples" is justifiable only when the action of, or reaction on, an axis is in question.

We hope that we shall not be misunderstood. Of course, Prof. Lamb is perfectly aware of all this; but our point is that, as the question is one of getting correct ideas into the minds of students, this venerable and "Through the Looking-Glass" mode of referring to these terms would be better forgotten. Students, and even some writers, are all too prone to suppose that the component growths of angular momentum are $A\dot{p}$, $B\dot{q}$, $C\dot{r}$, and the equations are naïvely written as $A\dot{p} = L$, etc.

It is, by the way, very important, and very easy, to take the terms $A\dot{p}$, $-Bqr$, Cqr , and show how each of them arises.

We miss here what is certainly very instructive: the application of the hodograph to the motion of a rigid body—for example, a top. A sequence of vectors all drawn from a point O is erected in space to represent the successive values of the resultant angular momentum. The curve in space on which lie their extremities is the hodograph for the rigid body motion, and the resultant couple at each instant is represented by the velocity of a point which moves so that it is always at the extremity of the vector which at the instant represents the angular momentum.

In connection with holonomous systems, the only reference is to Hertz, though the fact that there are systems which are not holonomous for which Lagrange's dynamical method "fails," and which require special treatment, was pointed out by Ferrers long before Hertz's treatise appeared.

The paper of Ferrers (*Q.J.M.*, vol. xii., 1873) appeared six years after the publication of Thomson and Tait's "Natural Philosophy," but it is curious that there is not in the second edition (1883) of that treatise any hint of the existence of the peculiar systems of which Ferrers gave examples. In his review of the second edition of the "Natural Philosophy" (*NATURE*, vol. xx., 1879), Clerk-Maxwell, in his own inimitable way, directed attention to the introduction in that work of the method of Lagrange. "The two northern wizards were the first who, without compunction or dread, uttered in their mother tongue the true and proper names of those dynamical concepts which the magicians of old were wont to invoke only by the aid of muttered symbols or inarticulate equations." The spell of the "northern wizards" was not free from defect, but either their good fortune or their instinct preserved them from the examples in which the use of an incantation insufficiently guarded in its terms might have led to disaster.

This has not been the fate of every wielder of the magic wand of Lagrange. In the first issue, about twenty years ago, of an important treatise on "Rational Mechanics" a discussion of the motion of a coin rolling and spinning on a horizontal table appeared, in which the method of Lagrange was used with erroneous results. The error led the distinguished author to the invention of a new method, in which a set of general dynamical equations, which could be used instead of those of Lagrange, and were applicable in all cases, was set up. The error thus had the fortunate effect of enriching dynamical science. It is not, alas! the fate of all who make mistakes to rise on the stepping-stones of their errors to higher and better things.

In our opinion, recourse is had to the method of Lagrange in far too many cases. The student flies to it on the appearance of the least difficulty. A proper training in dynamics, which should be experimental as well as mathematical, would give students the power of solving problems of all kinds by the direct application of first principles. The use of Lagrange's equations does not develop this—indeed, it has a directly contrary effect. In this power the dynamical students and graduates of our universities are sadly deficient.

A good account of Lagrange's method is followed in Prof. Lamb's book by an exposition of Hamilton's dynamical method, and this in its turn leads to Jacobi's discussion of the integration of Hamilton's canonical equations by means of the complete integral of Hamilton's partial differential equation fulfilled by his so-called principal function

S . The corresponding function S' connected with S by the relation $S + S' = \Sigma(pq)$ does not seem to be mentioned. A partial differential equation similar to, and yet curiously different from, that for S also holds for it. S is a function of the qs , t , and as many constants, which depend on the initial co-ordinates of the system, as there are independent co-ordinates; S' is a function of the ps , t , and the same number of constants as before, which, however, depend on the initial motion.

One or two examples of the solution of these partial differential equations—for example, Jacobi's discussion of the elliptic motion of a planet referred to three co-ordinates—would have added to the interest of what is in itself a very interesting chapter of an exceedingly interesting book.

A. GRAY.

Maya Civilisation.

The Inscriptions at Copan. By Sylvanus Griswold Morley. (Publication No. 219.) Pp. xii + 643 + 33 plates. (Washington: The Carnegie Institution of Washington, 1920.)

IN this large quarto volume of more than six hundred pages, efficiently illustrated, Dr. S. G. Morley has produced a work which may justly be regarded as of the highest importance. It is necessary to enforce this statement, almost with a feeling of shame, because there are so few individuals in this country who have the faintest idea what enormous strides have been made in the elucidation of Central American archæological problems since our own countryman, Dr. Maudslay, published the results of his explorations in a series of volumes, which Dr. Morley generously, but no less justly, describes as "easily the most important field contribution to Maya archæology." Those volumes might have been expected to give a lead to British archæologists and explorers; as a matter of fact, not only did no one appear in this country to carry on Maudslay's work, but the fine series of moulds of the principal Central American carvings, which he made at the cost of enormous labour and expense, lie buried in the cellars of the Victoria and Albert Museum, and not even an approximately representative series of casts taken from them is available to British students.

It follows that such a work as this is almost the despair of the reviewer. Most people have heard of the Aztec, but the earlier Maya civilisation is familiar to few even by name. Yet the Maya had evolved a remarkably fine art, an elaborate hieroglyphic script, and a very highly